MATH 54 – MIDTERM 3 STUDY GUIDE

PEYAM RYAN TABRIZIAN

Note: Midterm 3 is on **Friday, July 27th** in 4 Evans from 12:30 pm to 2 pm. However, **come early**, because we might start earlier than that! It covers sections $5.1 - 5.5$ of the Linear Algebra book and sections $9.4 - 9.8$ of the Differential Equations book. There will be 5 questions: 3 linear algebra ones, and 2 differential equations ones.

Note: 1.3.4 means 'Problem 4 in section 1.3'

CHAPTER 5: EIGENVALUES AND EIGENVECTORS

- Find all the eigenvalues of a matrix (you may have to use the rational roots theorem and long division to figure out what the roots are) (5.1.17, 5.2.3, 5.2.11)
- Given an eigenvalue λ, find all the eigenvectors corresponding to λ (5.1.9, 5.1.16)
- Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$ (5.3.7, 5.3.11, 5.3.17, try to do those without the hint on page 293)
- Use $A = PDP^{-1}$ to calculate A^3, $\sqrt{A}$, e^A etc. (5.3.1)
- Determine if a matrix A is diagonalizable or not (remember, you have to look at the eigenvectors, not the eigenvalues)
- Know examples of matrices which are not diagonalizable (see lecture), and know how to show that they are no diagonalizable
- Know that a square matrix is invertible if and only if 0 is not an eigenvalue of A (this is the last statement of the IMT)
- Show that if A is diagonalizable, then A^2, A^{-1}, A^T etc. are diagonalizable
- Show that if A and B are similar, then so are A^2 and B^2, A^{-1} and B^{-1}, A^T and B^T (5.2.24, 5.4.19, 5.4.20)
- Know how to find complex eigenvalues/eigenvectors (remember that you only need to do half of the work!) (5.5.1, 5.5.5)

CHAPTER 9: SYSTEMS OF DIFFERENTIAL EQUATIONS

Know how to:

- Write a system of differential equations in matrix form (9.4.1)
- Convert a higher-order differential equation (like $y''' + y'' - 2y' + y = 0$) into a system of differential equations (9.4.5)
- Solve a system of differential equations $\mathbf{x}' = A\mathbf{x}$ (9.5.12, 9.5.13, 9.5.14, 9.5.15, 9.5.19, 9.6.1, 9.6.3)
  Note: You do **NOT** need to know how to know how to draw pictures of the solutions you found!
- Show that 2 or 3 vector functions are linearly independent (9.4.15)

Date: Friday, July 27th, 2012.

- Solve systems using undetermined coefficients and/or variation of parameters (9.7.1, 9.7.3, 9.7.4, 9.7.11, 9.7.13, 9.7.15)
  Note: I will not ask you anything fancy, just know how to do the basic stuff! Also, if there's a complicated integral to evaluate, I will give you the formula!

  The following will **NOT** appear on the required portion of the exam, but might appear as a bonus question!

- Given A, know how to find e^A by diagonalizing A, and use this to solve $\mathbf{x}' = A\mathbf{x}$. Also remember the motivation leading to the definition of e^{At} (9.8.7)

TRUE/FALSE EXTRAVAGANZA

Check out the following set of T/F questions (solutions are in the HW hints, but beware, there might be mistakes, e-mail me whenever something seems to be wrong): 5.1.21, 5.1.22 (ignore (c)), 5.1.24 (not a T/F question, but good to know), 5.2.21, 5.3.21.

Also, review the T/F extravaganza that we covered in lecture, those are very good/important!

Note: There will be **NO** T/F questions about differential equations! However, there will be 5 T/F questions without justifications, and **2** T/F questions with justifications. They will *all* be linear algebra questions!

CONCEPTS

Here are a couple of concepts we learned so far. You **don't** have to memorize the definitions, just have a rough idea of what those things are

- Eigenvalue
- Eigenvector
- Characteristic polynomial
- Diagonalization
- Diagonalizable / Not diagonalizable
- The test where if a matrix A has n distinct eigenvalues, then it is diagonalizable
- Similar matrices
- System of differential equations
- Wronskian matrix / Fundamental matrix
- e^A